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The absolute normalization of spectroscopic factors for proton transfer reactions on ^{35,37}Cl

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Abstract. Spin-dependent sum rules for spectroscopic factors have been fitted to proton $s_{1/2}$ and $d_{3/2}$ spectroscopic factors obtained from (d, ^{3}He) and (^{3}He, d) reactions on ^{35,37}Cl. The fits with suggested spin assignments show deep minima in a plot of expected error in relative spectroscopic factors against absolute magnitude of the spectroscopic factors. Thus, if the relative spectroscopic factors have a 10% error, the absolute magnitudes are determined to about ±6%. The reduction in pick-up spectroscopic factors on ^{37}Cl found by analysing one set of experimental data is in agreement with more recent (d, ^{3}He) data. Theoretical wavefunctions for ^{35}Cl are examined in the light of the results obtained.

1. Introduction

Partial spin-dependent sum rules for spectroscopic factors were originally derived by French (1965) in a shell-model context. The fact that exact versions of the sum rules exist (Clement 1973) makes them an even more powerful tool in nuclear spectroscopy. They have been used to assign spins to nuclear states (Bjerregaard and Hansen 1967, Clement and Perez 1973) and the methods being developed by us enable tests of the accuracy of the absolute normalization of spectroscopic factors to be performed. It is this aspect we wish to stress in this application to proton transfers on ^{35,37}Cl although we are also able to assign a probable spin of 3^+ to the 7·33 MeV level of ^{36}Ar.

The analysis used is similar to that applied to $f_{7/2}$ neutron transfer on ^{45}Sc (Clement and Perez 1973). The constants n^+ and n^- renormalize the absolute magnitudes of the spectroscopic factors obtained from experiment using distorted-wave Born approximation (DWBA) analyses. For a given (jl) transfer on a spin-$\frac{3}{2}$ nucleus they are subject to the total sum rule

$$n^+ \sum_{J_n} S^+_{J_n} + \sum_{J_n} S_{cJ_n} + n^- \sum_{J_\alpha} S^-_{J_\alpha} = (2j+1)(1+2/A), \tag{1}$$

where $S^+_{J_n}$ and $S^-_{J_\alpha}$ are partial sums of the commonly used spectroscopic factors $\frac{1}{4}(2J_n+1)S'_n$ and S'_α (the primes denoting the inclusion of isospin Clebsch–Gordan coefficients) for stripping and pick-up reactions to states of final spins J_n and J_α respectively.

The factor $2/A$, where A is the target mass number, is the centre-of-mass correction for 2s–1d transfers (Clement 1973) and S_{cJ_n} is the unobserved continuum correction to the stripping which we neglect in the first instance.

The partial sum rules are expressed in terms of the quantities

$$Q_{J_n} = n^+ S^+_{J_n} + S_{cJ_n} + (2J_n+1)\left[\sum_{J_\alpha} \begin{Bmatrix} J_\alpha & j & \frac{3}{2} \\ J_n & j & \frac{3}{2} \end{Bmatrix} n^- S^-_{J_\alpha} - \frac{1}{4}\left(1+\frac{2}{A}\right)\right], \tag{2}$$

which vanish for a perfect fit.

To obtain a criterion for a fit we allow each individual spectroscopic factor to have a proportional error σ and calculate the quantities

$$q^2_{J_n} = \text{var}\, Q_{J_n}. \tag{3}$$

The errors on the individual spectroscopic factors rather than their sums (Clement and Perez 1973) are used because of the larger number of states associated with some final spins.

An estimate for σ is then obtained from the observed Q_{J_n} using the relation

$$\sum_{J_n} \frac{q^2_{J_n}}{2J_n+1} = \sum_{J_n} \frac{Q^2_{J_n}}{2J_n+1}. \tag{4}$$

This equation relates σ to n^- and n^+ which, if an assumption is made about the continuum contribution, are in turn related by (1). Thus for given n^+ and n^- the value of σ is an overall criterion of the goodness of fit. We might expect $\sigma = 0{\cdot}1$ corresponding to 10% errors in relative spectroscopic factors to be about the critical value above which fits should be rejected. An alternative criterion which we also quote is N, the number of partial sums with $Q^2_{J_n} > q^2_{J_n}$ when $\sigma = 0{\cdot}1$. Since there are two sets of experimental spectroscopic factors for two of the reactions considered we are also able to estimate their relative differences which provide 'experimental' values of σ.

We also discuss the limits on the amount of continuum strength in the sum rules and give a detailed comparison between experimental and theoretical spectroscopic occupancies in the case of ^{35}Cl.

2. Partial sum rule fits for ^{35,37}Cl

For both nuclei we assume that the low-lying $l = 2$ pick-up strength seen is entirely $d_{3/2}$. The assumption is supported by two observations, the first being that there is a gap of about 4 MeV between the two regions of $l = 2$ strength. Secondly the extensive shell-model calculations of Wildenthal *et al* (1971) (see their table 17) show little expected $d_{3/2}$–$d_{5/2}$ mixing for proton pick-up on ^{35}Cl. We thus have four partial sum rules for $d_{3/2}$ transfer and two for $2s_{1/2}$ transfer. The latter sum rules take a simple form and could be treated analytically (Clement 1973) but it is more convenient here to treat them in the same way as the $d_{3/2}$ sum rules.

2.1. Fits for ^{35}Cl

Spectroscopic factors, $\frac{1}{4}(2J_n+1)S'_n$ obtained from DWBA analyses of the stripping reaction (Moistener and Alford 1970) are given in table 1 together with known spin assignments (Endt and Van der Leun 1973) J_{obs} for ^{36}Ar. The small fragments, which mostly have angular distributions consistent with $l = 0+2$, have been lumped together into two 'states' with $J^\pi = 1^+$ and 2^+ respectively. Two pick-up experiments by

Table 1. Observed spectroscopic factors, $\frac{1}{4}(2J_n+1)S'_n$, for $l = 0$ and $l = 2$ transitions in $^{35}Cl(^3He,d)^{36}Ar$. J_{obs} are definite and (bracketed) likely spin assignments and J_{sr} are the most likely results of application of the partial sum rules.

State E(MeV)	J_{obs}	J_{sr}	$\frac{1}{4}(2J_n+1)S'_n$†	
			$l = 0$	$l = 2$
0	0^+	0^+		0·59
1·97	2^+	2^+		1·02
4·44	2^+	2^+		0·45
6·61	2^+	2^+		0·48
7·33	$(2^+, 3^+)$	3^+		0·20
Fragments	—	1^+	0·07	0·05
	—	2^+	0·07	0·05

† Moistener and Alford (1970).

Wildenthal and Newman (1968) and Puttaswamy and Yntema (1969) have been performed, and the corresponding spectroscopic factors, S'_α, and spins are listed in table 2. States above 4·5 MeV with $l = 2$ strength have been omitted as corresponding to $d_{5/2}$ transfer. Also the small $l = 0$ strength assigned by Wildenthal and Newman (1968) to states at 6·83 MeV and 7·11 MeV are omitted. Most of this strength may be spurious as the latter state is now thought to have spin 2^- (Endt and Van der Leun 1973).

Table 2. As for table 1 for $S'_\alpha(l)$ from two $^{35}Cl(d,^3He)^{34}S$ experiments.

State E(MeV)	J_{obs}	J_{sr}	Puttaswamy and Yntema (1969)		Wildenthal and Newman (1968)	
			$S'(0)$	$S'(2)$	$S'(0)$	$S'(2)$
0	0^+	0^+		1·00		0·86
2·13	2^+	2^+	0·25	0·37	0·21–0·26	<0·26
3·30	2^+	2^+	0·98		0·66–0·74	<0·52
4·07†	1^+					
		1^+	0·82		0·57–0·62	<0·28
4·11†	2^+					

† Unresolved doublet in these experiments.

Both experiments left the doublet at 4·07 and 4·11 MeV unresolved so that the observed pick-up strength can be assigned to either or both 1^+ and 2^+. Since the spectroscopic factors found by Wildenthal and Newman (1968) are much more uncertain, those of Puttaswamy and Yntema (1969) were used in the fitting procedure. The fact that there is actually good agreement between the two sets of spectroscopic factors is commented on later.

Table 3 shows the effect of varying n^- in the range 0·7 (0·1) 1·2 with n^+ determined by (1) with continuum contributions neglected. The sharp minimum in σ against n^- for $l = 2$ is shown in figure 1 together with the curve for $l = 0$. If we rely more on the greater number of sum rules for $l = 2$ and cut off at $\sigma = 0{\cdot}1$ the best value for n^- is

$$n^- = 0{\cdot}92 \pm 0{\cdot}06.$$

Table 3. Values of σ and N obtained from application of the partial sum rules to $l = 0$, 2 transitions in $^{35}Cl(^3He,d)^{36}Ar$ and $^{35}Cl(d,^3He)^{34}S$ when n^+ and n^- are changed subject to fitting the total sum rule.

Renormalization		$l = 0$		$l = 2$	
n^+	n^-	σ	N	σ	N
0·75	1·20	0·21	2	0·41	4
0·86	1·10	0·13	1	0·28	4
0·98	1·00	0·05	0	0·14	2
1·10	0·90	0·08	0	0·06	1
1·20	0·80	0·21	2	0·18	3
1·32	0·70	0·38	2	0·33	4

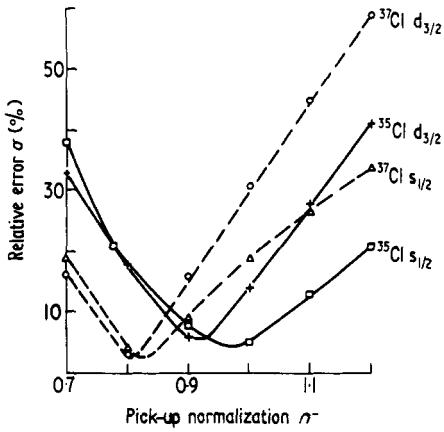

Figure 1. The expected error, σ, in relative spectroscopic factors as a function of the absolute renormalization constant, n^-, for pick-up spectroscopic factors for proton transfers on ^{35}Cl and ^{37}Cl.

This result would agree exactly with the assumption that ^{35}Cl has three protons in $s_{1/2}$–$d_{3/2}$ orbits which is supported by the absence of observed (fp) pick-up strength (Puttaswamy and Yntema 1969). For agreement between the two sets of pick-up strengths shown in table 2 we must have

$$n^-(d_{3/2}) = 0{\cdot}86, \qquad n^-(s_{1/2}) = 0{\cdot}70\text{–}0{\cdot}79,$$

where only the ground state $l = 2$ strength is considered.

The $l = 2$ result agrees essentially with the one we have obtained whilst there is some evidence that the $l = 0$ strength is underestimated by about 10% by Wildenthal and Newman (1968).

The above results were obtained for the spin assignments J_{sr} given in tables 1 and 2. A discussion of these and the effects of their variation is given in § 2.3.

2.2. *Fits for ^{37}Cl*

Two of the proton transfer experiments on ^{35}Cl (Moistener and Alford 1970, Puttaswamy and Yntema 1969) were also performed on ^{37}Cl. The renormalization constants n^+

and n^- are expected to be similar for these two cases, and this should provide a test of the overall consistency of the analysis. Tables 4 and 5 give the spectroscopic data which include a second pick-up experiment by Gray *et al* (1970) together with known spin assignments (Endt and Van der Leun 1973, Olness *et al* 1971) to the final states.

Table 4. As for table 1 for $\frac{1}{4}(2J_n+1)S'_n$ from $^{37}Cl(^3He,d)^{38}Ar$.

State E(MeV)	J_{obs}	J_{sr}	$\frac{1}{4}(2J_n+1)S'_n$†	
			$l=0$	$l=2$
0	0^+	0^+		0·47
2·18	2^+	2^+		2·47
3·94	2^+	2^+		0·12
Fragments	—	1^+	0·05	0·05
	—	2^+	0·05	0·05

† Moistener and Alford (1970).

Table 5. As for table 1 for $S'_a(l)$ from two $^{37}Cl(d,^3He)^{36}S$ experiments.

State E(MeV)	J_{obs}	J_{sr}	Puttaswamy and Yntema (1969)		Gray *et al* (1970)	
			$S'(0)$	$S'(2)$	$S'(0)$	$S'(2)$
0	0^+	0^+		1·31		1·06
3·31	2^+	2^+	1·21		0·86	
3·36	0^+					<0·10
4·52	1^+	1^+			0·75	
			1·24†			
4·58	2^+	2^+			0·25	

† Unresolved doublet in this experiment.

As for ^{35}Cl two 'states' with spins 1^+ and 2^+ are taken to share the small fragments of stripping strength observed. The pick-up data of Puttaswamy and Yntema (1969) was used in the sum rule analysis with the 4·52, 4·58 MeV doublet strength shared out in the ratio 3:1. In fact the sum rules analysis can 'resolve' the doublet as is described in the next section.

Table 6 shows the effect of varying n^- for both $l=0$ and $l=2$ transfer and again the sharp minima obtained in σ against n^- are shown in figure 1. At the $\sigma=0{\cdot}1$ level they lead to

$$n^- = 0{\cdot}81 \pm 0{\cdot}06.$$

To obtain agreement between the two sets of pick-up strengths shown in table 2 we would need

$$n^-(d_{3/2}) = 0{\cdot}81, \qquad n^-(s_{1/2}) = 0{\cdot}76.$$

Thus application of the sum rules predicts a reduction in strength observed in one experiment in agreement with a second experiment. The fit and conclusion remain

Table 6. As for table 3 for the $^{37}Cl(^3He,d)^{38}Ar$ and $^{37}Cl(d,^3He)^{36}S$ transitions.

Renormalization		$l = 0$		$l = 2$	
n^+	n^-	σ	N	σ	N
0·55	1·20	0·34	2	0·59	4
0·67	1·10	0·27	2	0·45	4
0·78	1·00	0·19	2	0·31	4
0·90	0·90	0·09	2	0·16	4
1·02	0·80	0·04	0	0·03	0
1·13	0·70	0·19	2	0·16	2

essentially unchanged on the addition of the 3·36 MeV 0^+ state with a spectroscopic factor of about 0·1.

The fit obtained is in agreement with the value $n^- = 0{\cdot}84$ based on three protons in $s_{1/2}$–$d_{3/2}$ orbits and is compatible with the value found for ^{35}Cl.

2.3. Sensitivity to spin assignments

The only significant spin ambiguities arising in the transfer experiments concern the doublets in ^{34}S and ^{36}S and the 7·33 MeV state in ^{36}Ar. The results of tests using the sum rules with fixed normalization constants are shown in table 7. In the cases of the two doublets the effect of allowing the normalization constants to vary does not change the conclusions.

Table 7. Sensitivity of σ to spin assignments to selected final states. The renormalization constants are $n^+ = 1{\cdot}10$, $n^- = 0{\cdot}90$ and $n^+ = 1{\cdot}02$, $n^- = 0{\cdot}80$ for proton transfers on ^{35}Cl and ^{37}Cl respectively. Except for the state considered the spins of final states are given by J_{sr} in tables 1, 2, 4 and 5.

Final nucleus	E(MeV)	Assumed spin	σ
^{36}Ar	7·33	$0^+, 1^+, 2^+, 3^+$	0·24, 0·12, 0·11, 0·06
^{36}S	4·52 + 4·58†	$1^+, 2^+, (1^+ + 2^+)$†	0·27, 0·86, 0·04†
^{34}S	4·07 + 4·11‡	$1^+, 2^+$‡	0·06, 0·99‡

† Doublet. The last entry corresponds to the S'_x observed by Puttaswamy and Yntema (1969) divided in the ratio 3:1 between the 1^+ and 2^+ states as observed by Gray *et al* (1970).
‡ Doublet. The entries correspond to the entire unresolved pick-up strength going to the 1^+ state or the 2^+ state.

The sum rule analysis suggests the entire $l = 0$ strength to the 4·07, 4·11 MeV doublet in ^{34}S goes to the 1^+ state and puts an upper limit of about 0·1 to the spectroscopic factor to the 2^+ member of the doublet. This result accords with shell-model predictions (Glaudemans *et al* 1964, Wildenthal and Newman 1968, Wildenthal *et al* 1971) that there is only substantial $l = 0$ pick-up strength to a 1^+ state in this energy region.

For the ^{36}S doublet which was unresolved by Puttaswamy and Yntema (1969) the analysis brings their result into line with that of Gray *et al* (1970). With the former

experiment alone we see that the assumptions that all the strength proceeded to 1^+ or 2^+ states alone are unacceptable according to our criterion on σ. In this sense the sum rule analysis can be said to resolve the doublet.

For the 7·33 MeV level in ^{36}Ar which is now known to be either 2^+ or 3^+ (Endt and Van der Leun 1973) the analysis suggests a 3^+ assignment. The 6·61 MeV level in ^{36}Ar is the analogue (Hardy *et al* 1969) of the ^{36}Cl 2^+ ground state, and thus the 7·33 MeV level is a likely candidate for the analogue of the ^{36}Cl 3^+ state at 0·79 MeV. This assignment is supported by a comparison of its spectroscopic factor in the proton stripping reaction (Moistener and Alford 1970), and those deduced from a shell-model calculation (Moistener and Alford 1970, Glaudemans *et al* 1964) and from the analogue neutron stripping experiment (Decowski 1971), which have values 0·20, 0·21 and 0·10, respectively. However, if n^+ and n^- are allowed to vary to 0·98 and 1·00, respectively, to reach a minimum σ for spin 2^+, the latter assignment is favoured. The sum rule analysis alone does not favour either spin.

2.4. Experimental relative errors

The two pairs of experimental sets of spectroscopic factors for the pick-up reaction provide an opportunity to estimate 'experimental' values of σ. As is implicitly assumed throughout this work, there can be systematic errors in absolute normalizations and possibly also between different (lj) transfer values. We should therefore only compare spectroscopic factors relative to their sum for a given (jl).

We adopt the simplest approach and define $P_n = S_n/\Sigma_n S_n$ and assume that

$$\text{var } P_n = \sigma^2 \bar{P}_n.$$

If there are N states an estimate of σ is then given by

$$\sigma^2 = \frac{1}{N}\sum_n \left(\frac{P_{1n}-P_{2n}}{P_{1n}+P_{2n}}\right)^2,$$

where P_{1n}, P_{2n} are the two measured values.

For $l = 0$ pick-up on ^{35}Cl to the three states given in table 2, use of the mean values of $S'(0)$ given by Wildenthal and Newman (1968) results in $\sigma \simeq 3\%$. If the ground state $l = 2$ transition is also included the figure rises to $\sigma = 5{\cdot}4\%$.

For the ^{37}Cl pick-up data given in table 5 we include the $l = 2$ ground state transition with the two $l = 0$ transitions to the 3·31 MeV state and the doublet to obtain $\sigma \simeq 3\%$.

The data are really too sparse to make reliable statistical tests but nevertheless give values of σ well under 10%.

2.5. Continuum contributions to the sum rules

The discussion hitherto has assumed no contributions, S_{cJ_n}, to the sum rules and we have found good fits to the sum rules for certain values of n^+ and n^-. We now wish to discuss how much unobserved continuum contributions there can be whilst maintaining these good fits.

The first point is that the fits are unaltered for interchange between $n^+S^+_{J_n}$ and S_{cJ_n}. We can reduce n^+ and put the missing strength in S_{cJ_n} as long as it is proportional to the observed $S^+_{J_n}$. For ^{35}Cl when n^+ is reduced to about the best fit value of n^- of

0·90 this implies continuum strengths of 6 % and 10·4 % for $2s_{1/2}$ and $1d_{3/2}$, respectively. For ^{37}Cl the corresponding figures are 3 % $2s_{1/2}$ and 15 % $1d_{3/2}$ continuum strengths when $n^+ \simeq 0{\cdot}80$. The missing strength can also be transferred from the pick-up strength but we do not think this is likely. It is fairly clear both from our fits and the other experiments that Puttaswamy and Yntema (1969) overestimated the pick-up strength. The latter is now in line with shell-model expectations and if much strength is missing we can ask the experimental question of where is the strength to replace it? From the pick-up experiments there is no evidence of $f_{7/2}$, $2p_{3/2}$ occupancy and from the stripping experiments evidence of only extremely small $d_{5/2}$ transfer to 4^+ states.

The assumption that S_{cJ_n} is proportional to the observed $S^+_{J_n}$ is highly arbitrary so that we have performed some tests with the opposite assumption that the $S^+_{J_n}$ are all equal. This happens to be highly insensitive for $l = 0$ transfer in the cases we consider where the shell is nearly full. At least 20 % of the strength can be transferred into the continuum without spoiling the fits. The results for expected error, σ, and number of sum rules, N, violating the 10 % margin are shown in table 8 for $d_{3/2}$ transfer. The fits are clearly significantly worse.

Table 8. Tests for continuum contributions to the $d_{3/2}$ sum rules equally divided between states of different J_n.

Nucleus	Continuum contribution	Renormalization		σ	N
		n^+	n^-		
^{35}Cl	10 %	0·91	0·90	0·13	3
	20 %	0·76	0·90	0·23	4
^{37}Cl	10 %	0·87	0·80	0·09	1
	20 %	0·74	0·80	0·19	3

To sum up, the situation regarding continuum strength is that, under increasingly restrictive assumptions about S_{cJ_n}, an increasing amount of observed strength can be transferred into the continuum by reducing n^+ and n^-. If the restrictions are lifted the fits become poor. We think a reasonable upper limit on continuum strength is about 10 %. With a figure of 20 % the fits we have obtained would be very fortuitous.

2.6. Comparison with theoretical wavefunctions for ^{35}Cl

The spectroscopic factors resulting from the sum rule analysis of experimental data can be compared to those resulting from shell-model calculations. If the errors we are trying to estimate can be taken seriously the process provides a severe test of shell-model wavefunctions.

For ^{35}Cl and ^{34}S shell-model calculations involving $2s_{1/2}$ and $1d_{3/2}$ orbits have been made by Glaudemans *et al* (1964) and the results of these and similar later calculations are reported by Wildenthal and Newman (1968). Also Wildenthal *et al* (1971) have performed more elaborate calculations which also involve the $1d_{5/2}$ orbits. We have chosen to compare the total proton $2s_{1/2}$ and $1d_{3/2}$ occupancies resulting from these calculations with experiment rather than spectroscopic factors for individual final states. This is because the occupancies, $S^-_{J_\alpha}$, are the quantities most closely specified by the sum rules from the corresponding stripping occupancies, $S^+_{J_n}$.

The comparison is shown in table 9 where the occupancies of Wildenthal *et al* (1971) are taken from their table 17. Some comments are necessary regarding the numbers given by this table. It was shown (Clement 1973) that the total s-shell occupancy, $n = S_1^-(0)+S_2^-(0)$, restricts the value of the ratio, $r_1 = S_1^-(0)/S_2^-(0)$, by the relation

$$\frac{3(n-1)}{3+n} \leqslant r_1 \leqslant \frac{1}{5}\left(\frac{5-n}{n-1}\right).$$

Table 9. Total single particle occupancies, $S_{J_\alpha}^-(l)$ and $S^- = \Sigma_{J_\alpha} S_{J_\alpha}^-(l)$, in ^{35}Cl, where $l = 0, 2$ correspond to the $2s_{1/2}$ and $1d_{3/2}$ orbits, respectively.

	Experiment		Theory			
	Puttaswamy and Yntema (1969)†	Wildenthal and Newman (1968)	Glaudemans *et al*		Wildenthal *et al* (1971)	
			(1964)	(1968)‡	FPSDI	MSDI§
$S_1^-(0)$	0.75 ± 0.05	0·57–0·62	0·43	0·46	0·90	0·86
$S_2^-(0)$	1.13 ± 0.07	1·14–1·27	1·08	0·98	0·93	1·01
$S^-(0)$	1.88 ± 0.12	1·71–1·89	1·51	1·44	1·80	1·87
$S_0^-(2)$	0.92 ± 0.08	0·86	0·78	0·78	0·98	0·96
$S_1^-(2)$	—	<0.28	—	—	0·11	0·12
$S_2^-(2)$	0.34 ± 0.02	<0.78	0·14	0·18	1·00	0·95
$S_3^-(2)$	—	—	—	—	0·15	0·16
$S^-(2)$	1.26 ± 0.08	0·86–1·92	0·92	0·96	2·24	2·19

† Experimental values renormalized with $n^- = 0.92 \pm 0.06$.
‡ Calculation reported in Wildenthal and Newman (1968).
§ Two calculations. See text for a discussion of the $2s_{1/2}$ strength.

The occupancies given by Wildenthal *et al* (1971) violate this relation principally because a relatively large pick-up spectroscopic factor to a 1^+ state between 6 and 7 MeV is predicted. No such strength is found experimentally. Furthermore the large value of $S_2^-(2)$ predicted is not observed experimentally and, in fact, would cause the sum rules from the stripping data to be badly violated.

Our conclusion is, therefore, that the wavefunctions predicted by Wildenthal *et al* (1971) have several features which disagree significantly with experiment. On the other hand the simpler wavefunctions of Glaudemans *et al* (1964) reproduce the data quite well, certainly as regards relative occupancies. This agreement extends to individual spectroscopic factors.

3. Conclusions

Successful fits have been made to the sum rules for $d_{3/2}$ and $2s_{1/2}$ proton transfers on ^{35}Cl and ^{37}Cl. A spin of 3^+ is predicted for the 7·33 MeV level in ^{36}A. The fits are quite sensitive to the renormalization, n^-, of the pick-up spectroscopic factors used and, at a level of 10% errors in relative spectroscopic factors, restrict the error in n^- to about

$\pm 6\%$. The suggested reductions in the spectroscopic factors of Puttaswamy and Yntema (1969) brings them into agreement with those of Wildenthal and Newman (1968) and Gray *et al* (1970). Estimates show that the relative spectroscopic factors obtained from these experiments differ by substantially less than 10%.

As found in earlier work (Clement and Perez 1973) we would find it difficult to maintain the good fits to the sum rules with more than 10% of the single-particle strength in the continuum. The conclusions of our earlier work regarding this point are therefore reinforced.

Since we now have greater reason to rely on the occupancies, $S_{J_\alpha}^-$, obtained for the ground state wavefunctions the detailed comparison with theoretical wavefunctions for ^{35}Cl may be valuable. We have no understanding of why the more elaborate shell-model wavefunctions of Wildenthal *et al* (1971) appear to do worse than the simpler wavefunctions of Glaudemans *et al* (1964).

The usefulness of the sum rules in this part of the periodic tables points the way towards further applications. The nucleus ^{33}S would be a possibility but proton pick-up reactions have not yet been performed on it.

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